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Qualitas and Quantitas: two ways of thinking in science

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Abstract The paper explores the complex history of quality and quantity from Aristotle's doctrine of categories up to current discussions of the status of *qualia* in the mind-body problem in modern analytic philosophy. In the first part of the paper we trace the progressive mechanisation, mathematisation and quantification of the natural sciences, processes which spread to the humanities and medicine as early as the seventeenth and eighteenth centuries and later culminated in the logical positivism of the Vienna and Berlin Circle. The second part discusses the renaissance of qualitative research methods in the nineteenth and twentieth centuries in the humanities and the social sciences (hermeneutics, descriptive psychology, phenomenological sociology; the Chicago and Frankfurt School). It describes the origin of the terms "quality of life", "quality of the environment", "qualitative growth", etc., and brings out the quite unexpected fact that qualitative research methods are nowadays also in vogue in mathematics, physics and artificial intelligence. The paper is based on a comprehensive search of the databases of several libraries via the keywords "quality" and "quantity". It contains a bibliography of some 200 items.

Keywords History and philosophy of science · Quantitative and qualitative research methods · Quantitative-qualitative debate · Logical positivism · Sustainable development

One has only to take a closer look at contemporary public debates to come across the seemingly outdated antagonism between quality and quantity. For example one might encounter the urgent call for qualitative growth instead of an undifferentiated quantitative one in order to give our planet a chance to survive: pure quantitative growth of industrial production or a gross national product is no longer needed, but "qualitative" or (more recently) "sustainable growth", i.e., an increase in the quality of life has to be our prime concern.

The confrontation between quality and quantity being used in countless contexts has become an almost meaningless catchword. There is talk of the quality and quantity of lamb

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production in western Anatolia (Wassmuth and Sarican 1984) or about the quality and quantity of tanks (Stark 1982) or about the tension between the quantity of students and the quality of education at universities (Radford et al. 1997). It is a striking fact that in public opinion the notion of quality is associated with attributes such as warm, humane, holistic and concrete, whereas quantity, or quantification, is often connected with cold, technocratic, isolating, reductionist, abstract, and is thus valued rather negatively. This preference for quality goes along with the fact that quality does not mean just a defining property as it did in antiquity, but the aspired positive attribute, that is the "goodness" or excellence of a thing (Pirsig 1974/1999).

In a noteworthy contrast to the described public opinion, contemporary science is still predominantly quantitatively oriented—a fact that has often been criticised. The prototypical scientist, e.g., the engineer as portrayed by the Swiss author Max Frisch in his novel *Homo Faber* (1957), tends to value *qualitas* rather negatively. For him quality is something non-scientific, soft, not easy to comprehend, actually something that does not even deserve scientific treatment. In today's scientific community the opinion is still predominant that quantifiability of the phenomena is a precondition for a scientific approach. The comprehensive and programmatic quantification of scientific methods, which has left hardly any scientific discipline untouched, is a relatively young phenomenon: it only started in the early modern period and perhaps it even marks the beginning of this period. How did all this happen? What are the reasons? This is the historic question that I want to treat in the following pages.

Of course on closer examination the relation between *quantitas* and *qualitas* is far from being simple and antagonistic, as it is often portrayed in public. The discussions concerning this matter show clearly that, if there is talk of "quality" in the above-mentioned and similar contexts, there is mostly an overriding interest in quantification involved, that is in the measurement of these qualities, especially when they are the issue in a political debate. For instance, in the discussion concerning air quality, the method of measurement, that is, the determination of indicators and limit values, plays an important role; and in debates on the quality of life, quantities are an important issue; they include suicide rates or financial resources that are made available in order to build noise barriers or to establish permanent posts for teachers and social workers. Even the promoters of alternative, qualitatively oriented concepts accept the assumption that there is a connection between these quantities and the desired quality improvement, namely one of cause and effect.

A detailed discussion of the complex, multi-layered issue "science between quality and quantity" seems rewarding to me, the more so as it raises further basic questions, which since their first systematic study by Plato and Aristotle have not lost their current relevance. Are there any limits to quantification, which since the Middle Ages and the Modern Era has come to embrace ever wider areas, and to the latest digitalisation of "qualities"? Will the exact sciences in the future intensify the use of qualitative notions and methods, as has happened in the humanities during recent decades? Can the necessary interdisciplinarity between the humanities and the natural sciences be established, or will the quantitative remain the primary realm of the natural sciences and mathematics, while the qualitative is restricted to the social sciences and humanities? What are the causes for our "addiction to quantification", which since the nineteenth century encompasses almost every area of life? What is it that makes the quantitative understanding of the world so attractive? Is it really the most precise understanding of individual phenomena, or is it rather the desire to pass on a multitude of interrelated factors to other people in a simple and unassailable way? Don't we thereby often accept a loss of the essential properties of a thing, as for example the so-called *qualitas occulta* or *qualitas virtualis* of the medieval scholastics?

Of course it will not be possible to discuss or, still less, answer all of these questions within this short survey. Yet in the following I would like to sketch out the historical development and the astonishing "renaissance of the qualitative", which is evident in many subject areas. At the same time I take the opportunity of introducing our German collected volume *Wissenschaft zwischen Qualitas und Quantitas* (2003) to an English-speaking public. Here you will find specific contributions on different questions of the whole subject, including a comprehensive bibliography (about 500 titles up to 1995), which I have both reduced to the most important publications and updated for this contribution.[1]

1 From Qualitas to Quantitas—from antiquity to the modern era

The central position of the contrasting notions "Qualitas–Quantitas" dates back to antiquity, especially to Aristotle. In his doctrine of categories both notions belong to the basic forms of proposition about the being, together with substance, relation, place, time, position, state, action and affection. According to Aristotle "quantity" "means that which is divisible into two or more constituent parts of which each is by nature a 'one' and a 'this'."[2] Quantities are either discrete or continuous and they have no contraries, they do not admit of a variation of degree, and they are called equal and unequal.[3] In his *Categories* Aristotle defines "quality" as follows: "By quality I mean that in virtue of which people are said to be such and such".[4] In the following he distinguishes between four different sorts of qualities: 1. habit or disposition; 2. inborn capacity or incapacity; 3. affective qualities; 4. figure and shape. All the qualities are to be compared in terms of similarity.

In his books on natural science Aristotle built the structure of the universe on the Empedoclean four elements (earth, water, air, fire) and the seven pairs of haptic qualities of sense-perception (hot–cold, moist–dry, heavy–light, coarse–fine, rough–smooth, hard–soft, viscous–brittle). Each of the four elements has its natural place; the earth at the centre of the universe, then water, air, fire, arranged in concentric spherical shells, followed by the spheres of the moon and the six other planets (Mercury, Venus, Sun, Mars, Jupiter, Saturn) and finally the sphere of the fixed stars. In the sublunar world Aristotle makes a distinction between natural and violent motion. Natural motion is a result of the innate tendency of the predominant element in a body to move in a straight line towards its natural place and to come to rest there [for example the free fall of a stone (predominant element: earth) towards the middle of the earth, or the ascent of smoke towards the region of fire]. Every motion ("motion" in Aristotle stands for all kinds of change: alteration of quantity, quality or place, instantaneous change) is due to a mover. For the natural change of place the mover is the *generans* that generated the bodies, that is to say their heaviness or lightness. Thus Aristotle's world view was to a large extent a qualitative one, in so far as in his theory the behaviour of the bodies was determined by their qualities.[5]

In later antiquity and in the Middle Ages Aristotle's theories were thoroughly discussed and refined in several subject areas. Important innovations in the field of motion were the development of an impetus theory in order to explain violent motion by the Alexandrian

[1] In the following footnotes I will refer for each subject area to the relevant publications of the comprehensive bibliography in Neuenschwander (2003) and list the most important new contributions. Copies of Neuenschwander (2003) are still available in the book trade or by the author.

[2] Aristotle, *Metaphysics*, book V, Chap. 13 (1020a).

[3] Aristotle, *Categories*, Chap. 6 (4b).

[4] Aristotle, *Categories*, Chap. 8 (8b).

[5] For further information on Aristotle's world picture see Dijksterhuis (1950), Grant (1977), Lang (1992), Pedersen and Pihl (1974).

Johannes Philoponus, the Arabs and Johannes Buridanus as well as the subsequent system of latitude of forms (*latitudines formarum*). By means of the latter, medieval scholastics managed for the first time to comprehend quantitatively the intensive increase or decrease of accidental forms (*intensio et remissio formarum*) or qualities. In his *Tractatus de configurationibus qualitatum et motuum* Nicole Oresme (about 1322–1382) expanded the idea of latitude of forms to a complete system, in which qualities and velocities have at least two dimensions, namely intensity and expansion in space or time. In spite of outstanding results, which were developed further by Galileo Galilei, the scholastic theory of latitude of forms was still oriented largely qualitatively and also somewhat speculatively.[6]

In the beginning of the Modern Era the theories of the Middle Ages were increasingly criticised and amended by new-found knowledge. For example, Renaissance mathematicians developed an algebraic symbolism by means of which functional dependencies could be described algebraically, not only geometrically as shown by Oresme. René Descartes (1596–1650) assembled these two basic approaches in his *Géométrie*, and so analytic geometry was born, which was an essential precondition for the development of Newtonian theories. In his *Regulae ad directionem ingenii* Descartes planned to present a universal method for the solution of problems: every kind of problem has to be reduced to a mathematical problem, and this reduced again to an algebraic problem and finally to the solution of a single equation (Pólya 1962–1965; vol. 1, p. 22). A similar view, which goes back to the Pythagoreans and Plato, was held by Galileo Galilei in his *Saggiatore*. There he stated that the book of nature is written in the language of mathematics; its letters are the triangles, circles and other geometrical objects, and without the knowledge of these it is impossible for human beings to understand a single word of it.[7]

The belief that every process in nature can be described in mathematical terms, is based on the quantifiability of the phenomena. That is why the period between 1543 and 1687 is generally regarded as marking the beginning of modern science. It begins with the publication of Copernicus' *De revolutionibus orbium coelestium*, which for the first time effectively questioned the old Ptolemaic world view, and it ends with Newton's *Philosophiae naturalis principia mathematica*, which served thereafter as the basis for the New Physics. Copernicus had formulated his heliocentric theory in what was still a quite philosophical way, lacking new astronomical observations and based on inherited data and conceptions. The revolutionary modern paradigm of scientific research was revealed in its full force only later with Tycho Brahe (1546–1601), Johannes Kepler (1571–1630) and Galileo Galilei (1564–1642). Decisive elements in this were the comprehensive and, thanks to improved instruments, significantly more precise data that Brahe had gathered during his 20 years at his astronomical observatory, Uraniborg, on the island of Hven. It compelled Brahe's assistant Johannes Kepler, after repeated comparison between theory and observation, to finally conclude in about 1605, that the orbit of Mars—contrary to the Platonic-Aristotelian axiom, which had been accepted since antiquity—cannot be described by means of circular movements with constant angular velocity, but must in fact be a non-uniform elliptic motion, with the sun placed at one focus of the ellipse. This led Kepler to the formulation of his famous laws, which later formed a foundation for Newton's laws of motion. Experiment, i.e., measurements and hence quantities, also played a decisive role when Galileo verified his new law of fall; it led him and his disciples to a mathematical description of motion, to a first theory of gravity, to the notion of inertia, and hence they "revolutionised" Aristotelian physics.

[6] For more details on the further development of Aristotle's theories in late antiquity and the Middle Ages see also Clagett (1959, 1968), Sylla (1971/1972, 1991), and Wolff (1978).

[7] Opere di Galileo Galilei, Edizione nazionale, 20 vols., Firenze 1890–1909; vol. 6, p. 232.

Most scholars of the Modern Era emphatically rejected the generalised peripatetic doctrine of qualities, which was passed down by the scholastics. Based on the corpuscular atomism of antiquity and the Middle Ages, Galilei already stated that sensory qualities such as colour, taste and odour do not reside as such in their respective objects, that is outside the observer, but that sensual perceptions are caused only by the way atoms and their movements affect our senses. Similar explanations were given by Descartes, Hobbes, Boyle and Locke, who all believed that sensory qualities are not necessarily objectively existing entities, but are merely subjective phenomena, and are hence to be traced back to the real spacial movements of the "atoms". As a result, a fundamental distinction was soon made between objectively existing primary qualities (such as shape, extension, number), which were thought to belong essentially to a certain object, and the merely subjective secondary qualities (such as colour, taste, smell). Thus Aristotle's categorical separation of quantity and quality became permeable and as a consequence vulnerable.[8]

The developments I have sketched led to a progressive mechanisation, mathematisation and quantification of natural science (Dijksterhuis 1950; Woolf 1961; Shea 1983; Frängsmyr et al. 1990). Already in the seventeenth and eighteenth centuries it spread to the humanities and medicine, as is shown by William Petty's *Political Arithmetick* (1690), William Harvey's *De motu cordis* (1628) and Lamettrie's *L'homme machine* (1748). Condorcet (1743–1794), who coined the notion "social science" ("science sociale"), already held the view that progress in his field could only be achieved if the rigorous methods of mathematical calculation and the science of combinatorics ("science des combinaisons") were applied (Baker 1975, p. 333). And Adolphe Quetelet (1796–1874) even stated that as they progressed all the sciences would be brought closer and closer to mathematics, as a kind of centre, to which all of them converged.[9] From the seventeenth century this view was quite common, as the writings of Descartes, Leibniz, Fontenelle, Wolff, Lambert, Laplace and countless others show. The same point is made by Kant, who wrote in the preface of his *Metaphysical Foundations of Natural Science* (1786), that "in every special doctrine of nature, only so much science proper can be found as there is mathematics in it."[10]

Towards the end of the eighteenth century parts of chemistry could be described atomistically by means of the newly found doctrine of chemical elements, and physics advanced to the position of a supposedly completed paradigmatic leading science in the nineteenth century. Influenced by positivism and evolutionary thinking, scholars of most of the social sciences tried to put their fields on a physical and mathematical basis: examples include sociology (statistical analysis, sociometry), political economy (econometry), political science (opinion polls), and psychology (psychophysics, behaviourism, psychological tests, etc.). Numerous scholars, such as Fechner, Du Bois-Reymond, Helmholtz, Kirchhoff, Mach, Boltzmann, Frege, Poincaré, Pearson, D'Arcy Thompson, Duhem, Russell and Einstein subsequently adhered more and more firmly to the view that every science is to be built on logical and physical foundations rather than on metaphysical theories, since this was the only way of establishing a language that would be intersubjectively comprehensible and apply to any matter of fact. Ludwig Wittgenstein wrote in his influential *Tractatus logico-philosophicus* (1921/1922): "The totality of true propositions is the total natural science (or the totality of the natural sciences)" (TLP 4.11). Even more trenchantly, Rudolf Carnap stated some years

[8] See S. Blasche in Ritter and Gründer (1971–2007, vol. 7, column 1766 ff.) For further information on the rise of the "New Science" and the Copernican Revolution see Neuenschwander (2003, p. 27, notes 7–9).

[9] Quetelet (1828, p. 232 f.)

[10] *Kants gesammelte Schriften*. Erste Abtheilung, Werke, Bd. 4, Berlin 1911, S. 470. For an extensive discussion of the consequences of this mathematisation, see the publications listed in Neuenschwander (2003, p. 28, note 11).

later that science was the system of intersubjectively valid sentences, and the language of physics was the universal language of science.[11] Working from these basic concepts, Logical Positivists of the Vienna Circle (M. Schlick, O. Neurath, R. Carnap) and the Berlin Group (H. Reichenbach, W. Dubislav, K. Grelling, A. Herzberg) fashioned an influential philosophical system. But they tended to disregard the fact that their notion of exactitude, especially in the social sciences and the humanities, went along with a notable narrowing and impoverishment of the objective. Of course this could not be accepted without contradiction by contemporary epistemologists, as is shown by the subsequent papers of Popper, Kuhn, Lakatos, Feyerabend and Quine.[12]

2 The "renaissance of the qualitative"

Despite the dominance of the quantitative since the early Modern Era there have been scientists in all periods who have stressed the importance of qualitative, holistically oriented studies; examples include Leibniz or Goethe in his work on colour theory. Amongst the most important roots of today's "renaissance of the qualitative" in the humanities and social sciences are historicism and hermeneutics, which arose in the second half of the nineteenth century through the work of Leopold von Ranke (1795–1886) and Wilhelm Dilthey (1833–1911). Dilthey attacked Auguste Comte's positivistic sociology and presented hermeneutics and descriptive psychology as the corner-stones of his own "Human Sciences", just as mathematics was the corner-stone of natural science. Descriptive psychology, according to Dilthey, proceeds from the object, from direct experience of the "psychic nexus" (seelischer Zusammenhang) and not, as in the natural sciences, from preformulated hypotheses. Other "qualitative"—as they are called today—theories are partly influenced by Dilthey, for example, Max Weber's (1864–1920) "understanding sociology", Edmund Husserl's (1859–1938) "phenomenology", and Alfred Schütz's (1899–1959) "phenomenological sociology", as well as the anthropological field studies of Franz Boas and Bronislaw Malinowski, who again decisively influenced the famous Chicago School of sociology.

The tension between *qualitas* and *quantitas* appeared in different ways within the particular fields of the social sciences and in each country, so only a few central aspects can be elaborated here. Already towards the end of the nineteenth century an embittered "Methodenstreit" (methodical dispute) emerged between qualitative-historically and quantitative-theoretically oriented scientists (Bryant 1985, pp. 57–108). From 1883 on, the well-known Vienna economist Carl Menger published several papers in which he laid out the justification and the necessity of axiomatic-deductive research and simultaneously attacked Gustav Schmoller's historical school, which was dominant in Germany at this time. Menger's ideas later gained growing support. Together with the successive development and spread of statistical methods (F. Galton, K. Pearson, G. U. Yule, R. A. Fisher) as well as numerous similar efforts from the positivistic side [for example under O. Neurath and P. F. Lazarsfeld (Halfpenny 1982, pp. 57–61)] they also stimulated a marked increase of quantitative research in the social sciences, whose share in the USA rose between 1895 and 1965 from 14 to 69% according to a study by Patel (1972, p. 6). Quantitative studies in election polls already had a long tradition in the USA (Literary Digest, Gallup Institute) and in the course of the commercialisation of

[11] Carnap (1931, especially p. 448). English translation *The Unity of Science*, London: Kegan Paul, 1934.

[12] For further information on these theories and the subsequent developments see Bryant (1985), Dahms (1994), Halfpenny (1982), Halfpenny and McMylor (1994), Kern (1982), Lancaster (1994), Porter (1995), Rescher (1985), Schulte and McGuinness (1992), Stadler (1993), Tolman (1992), Tuschling and Rischmüller (1983), as well as Neuenschwander (2003, p. 28, note 13).

social research they developed into a veritable "polling industry", which also spread out to the universities and led to the establishment of quantitative opinion research and empirical sociology (Kern 1982, p. 190 ff.)

A safe haven for qualitative methods in the twentieth century was provided at first by the Chicago School and later by the Frankfurt School. In the Department of Sociology of the University of Chicago, under the direction of William I. Thomas, Robert E. Park and Herbert Blumer from 1920 to 1940 individual case studies in the sense of Malinowski were highly valued, in contrast with the favour shown at Columbia University for mostly statistical studies (Faris 1967; Bogdan and Biklen 1982/2006; Bulmer 1984; Hammersley 1989; Guth 2004). The numerous staff members and graduates of the Chicago School spread qualitative research methods all over the USA, and together with the widespread social criticism in the sixties and seventies this led to a renaissance of qualitative studies (Glaser and Strauss 1967, p. 12 ff.). It gradually made its mark on most disciplines in the social sciences (sociology, social and cultural anthropology, ethnology, social psychology, economics, political science) and later on some neighbouring disciplines [pedagogy, psychology, sociolinguistics, social geography, history (oral history, microhistory)]. It moved on to Germany, where the Frankfurt School (M. Horkheimer, Th. W. Adorno, J. Habermas) (Sahner 1982; Bottomore 1984) had just effectively fought the "Positivismusstreit" (Adorno et al. 1969; Dahms 1994), and after intense methodological debate led in the eighties to a consolidation and demarcation of the qualitative-interpretative approach in opposition to the traditional quantitative-normative approach (Witzel 1982, p. 7; Garz and Kraimer 1991, p. 4 ff.; Zedler and Moser 1983, p. 7 ff.). Today the two approaches are regarded as complementary rather than competing concepts in social science research, as is evident from the numerous methodical comparisons, the descriptions in textbooks and the still growing amount of literature in the field of the qualitative.

The best known fields of study of the new so-called *qualitative social research* are, according to Bässler (1987) and Witzel (1982), Albert Schütz's phenomenological sociology and Harold Garfinkel's ethnomethodology, as well as George Herbert Mead's and Herbert Blumer's symbolic interactionism. The most important methods and techniques are according to Lamnek (1988/1989) the individual case study, the qualitative interview, the group discussion, the qualitative analysis of content, participant observation and the biographical method. They are all aimed at comprehending the counterpart as a whole and an individual, and they decidedly dismiss the standardised instruments of quantitative research (scales, tests, questionnaires), which deprive their "test persons" of any freedom of action and individuality. According to Cook and Reichardt (Cook and Reichardt 1979, p. 10 ff.), qualitative research is generally phenomenological, observational, subjective, descriptive, process-oriented, ungeneralisable, holistic, etc., whereas quantitative research can be characterised by such attributes as positivistic, measurable, objective, hypothetico-deductive, outcome-oriented, generalisable, particularistic etc. During the last two decades a continuously rising tide of publications concerning qualitative research has appeared, on which unfortunately we can only report in a very summary overview owing to lack of space. There are several book series and journals in this subject area (*Qualitative Sociology* 1978/1979 ff., *Qualitative Research Methods* 1986 ff., *QSE* 1988 ff., *Qualitative Inquiry* 1995 ff., etc.), a number of handbooks and collections of source material (Denzin and Lincoln 1994/2005; Bryman and Burgess 1999/2007; Denzin and Lincoln 2001; Boudon et al. 2003; etc.) and numerous research methods textbooks, some of them now reaching ten editions (Bogdan/Biklen; Flick; Mayring; etc.).[13]

[13] For a methodological discussion of the qualitative and quantitative approaches, the paradigma wars in the eighties and the history of qualitative research see Bogdan and Biklen (1982/2006), Boudon et al. (2003), Bryman (1988), Denzin and Lincoln (2001), Gage (1989), Glassner and Moreno (1989), Hammersley (2008),

The startling renaissance of the qualitative in the social sciences is hardly explicable without the preceding phase of rapid economic growth, with its partly negative consequences (environmental pollution, affluent society) and the wide-spread unrest about the political and social situation (Vietnam war, student riots). This lead to a more global, holistic world view and influenced the humanities as well as the natural sciences. It was probably nurtured by the development of the environmental sciences in which some of the basic so-called qualitative notions (quality of life, environmental quality, qualitative growth, etc.) played a vital role, as is to be shown in the following.

The notion "quality of life" was already used by about 1920 by A. C. Pigou (Noll 1982, p. 9); with its current meaning it was probably coined by the American TV commentator E. Sevareid, who used it to characterise A. Stevenson's election manifesto in 1956. Soon after it was popularised by the later Kennedy advisers A. M. Schlesinger Jr. and J. K. Galbraith—the author of the influential work *The Affluent Society* (1958)—it was then picked up by J. F. Kennedy himself in his 1963 State of the Union Address, when he said: "The quality of American life must keep pace with the quantity of American goods." The idea of quality of life showed up thereafter in several other presidential addresses, e.g., in those of Johnson and Nixon. Finally it spread worldwide through the works of J.W. Forrester and the Meadows in connection with the first report on *The Limits to Growth* of the Club of Rome (1972). In the German-speaking area the notion of quality of life was probably introduced in a speech by W. Brandt in 1971. In 1972 it was made part of the slogan in Sozialdemokratische Partei Deutschlands (SPDs) election campaign and went on to become a political catchword.[14]

The notion "quality of the environment" shows up already in the 60s in the titles of two books (Herfindahl and Kneese, *Quality of the Environment*, 1965; Jarrett, *Environmental Quality in a Growing Economy*, 1966). Following the economic boom of the 50s and 60s, people increasingly emphasised that our natural environment is being damaged irreversibly by an uncontrolled population explosion, unbridled economic growth and uninhibited consumerism (greenhouse effect, ozone hole, soil erosion, air pollution). In response, there were calls for methods and laws in order to lessen the pressure on the environment (improvement or preservation of air, water and soil quality), and to restrict the ruinous exploitation of natural resources (minerals, fossil fuels, tropical forests, diversity of species, natural environment, etc.) as well as a comprehensive system of waste management. All of these required a restriction or decrease in the destructive components of economic growth, a consequence that led to the newly coined notion "qualitative growth". As is generally known, *qualitative growth* is an alternative to zero growth, which in business circles was always considered as unrealistic and impracticable. Both suggestions were made in the sequel to the report of the Club of Rome and discussed by Mishan, Ehrlich, Forrester and in innumerable other publications and governmental reports. The entire array of questions is currently discussed under the notion "sustainability".

It is a difficult task to define or quantify notions like "quality of life". For this purpose, the relevant components of the notion must first be identified (for example health, opportunities for personal fulfilment, quality of working life, material happiness, social and physical

Footnote 13 continued
Rizo (1991), Salomon (1991), Smith (2003/2008, Chap. 2); for more information about the extensive literature in the various fields of qualitative research see Neuenschwander (2003, p. 28 f., notes 14–16).

[14] For further information on the conceptual history of the notion "quality of life" see H. Holzhey in Ritter and Gründer (1971–2007, vol. 5, column 141 ff.), C. Amery in Schultz (1975, p. 8 ff.), and W. Glatzer in Seifert (1992, p. 51 f.) For substantial discussion see Campbell et al. (1976), Friedrichs (1973–1974), Schultz (1975), Seifert (1992), Szalai and Andrews (1980) and the bibliographies of Agarwal et al. (1976) and Merwin (1976).

environment, security of life and property), which then have to be operationalised by means of one or several social indicators (the availability of medical care, college graduation rate, unemployment rate, housing density, population density) and finally weighted according to importance and significance. It is hardly surprising that, depending on the underlying model of quantification, the results differ greatly, which shows in turn how difficult it is to discover and eliminate the numerous subjective elements that enter in any attempt to quantify qualities.[15]

Another important though less known aspect of the renaissance of the qualitative is the development of so-called qualitative methods and fields of study in mathematics and theoretical physics. Qualitative considerations have a long tradition in mathematics, as in most natural sciences (chemistry, biology, etc.), but they came to be marginalised as quantification advanced. A short look at today's mathematical databases, however, reveals the unexpected fact that so-called qualitative theories in mathematics are flourishing again: In the last 20 years far more than a thousand articles with the term "qualitative" in their titles have been published. These developments have been outlined in detail for the first time in the introduction of our book *Wissenschaft zwischen Qualitas und Quantitas* (2003).

Apart from philosophically oriented studies by Leibniz and Hegel, the first basic approaches towards qualitative methods in mathematics were not made until the second half of the nineteenth century. One area in which these initially occurred was topology (by then usually called *Analysis situs*), notably in the work of C. F. Gauss, B. Riemann and J. B. Listing. Another was the subsequent formation of the so-called *qualitative theory of differential equations* and the *qualitative theory of dynamical systems*. The latter are connected with the names of H. Poincaré, A. M. Lyapunov and I. Bendixson and they owe their origins to the great difficulties that these mathematicians dealt with in treating dynamical systems in celestial mechanics and other fields. For example, the system of differential equations, which emerges from the n-body problem, can be "integrated" for $n \geq 3$ only in exceptional cases in finite terms; hence towards the end of the nineteenth century mathematicians were looking for new methods for making statements about the behaviour of such systems without knowing their explicit solutions. They wanted to describe the solution, if not exactly and comprehensively then at least approximately in its most relevant aspects, in other words "qualitatively". In this context Poincaré noted in his path-breaking *Mémoire sur les courbes définies par une équation différentielle* (1881–1886) that the study of differential equations can be divided into two sections, namely the qualitative, that is the geometrical study of the solutions, on the one hand, and their subsequent numeric calculation, on the other. Poincaré emphasised the utmost importance of such qualitative studies, because they make it possible to decide whether certain dynamical systems in celestial mechanics (for example our planetary system) behave in a stable or unstable manner in the course of time. He refined his methods in his monumental work *Les méthodes nouvelles de la mécanique céleste* (3 vols., 1892–1899) and presented them in 1908 in his plenary lecture at the International Congress of Mathematicians in Rome. This encouraged several other mathematicians, including Lyapunov, Picard, Hadamard, Levi-Civita and Bendixson, to continue with research in this field. As a result, by 1931 M. Petrovitch was already able to summarise their achievements in his monograph *Intégration qualitative des équations différentielles*, a pioneering work that has sadly been ignored by most historians of mathematics.

Since then, Soviet mathematicians and physicists in particular have pushed ahead with the qualitative theory of differential equations and dynamical systems. Crucial in this pro-

[15] For supplementary information on the problem area "quality of life, environmental quality, qualitative growth" see Neuenschwander (2003, p. 30, note 20) and Pretty et al. (2007).

cess was the impetus given by the work of Andronov (1929), who for the first time applied Poincaré's theory to the non-linear differential equation for the triode oscillator, that had been studied by B. van der Pol. Andronov also pointed out the relations between van der Pol's graphical solution and the limit cycles in Poincaré's theory. After the connection between Poincaré's theory and the theory of non-linear oscillations was established, these relations were systematically analysed by the Moscow research group Automation and Remote Control in the 1930s (Bissell 1998; Israel 2004). Together with numerous subsequent studies by other Soviet mathematicians and physicists, the results were published successively in textbooks and monographs on the qualitative theory of differential equations and non-linear oscillations; examples are the well-known works of Andronov, Bogolyubov, Krylov, Mandelstam, Nemytski, Pontryagin and Stepanov. Initially in the west relatively few researchers, apart from G. D. Birkhoff, tackled the problems initiated by Poincaré, until S. Lefschetz began to take an interest and went on, over a period of some years, to translate several fundamental Russian works into English (for example Kryloff and Bogoliuboff 1943; Andronov and Chaikin 1949; Nemytskii and Stepanov 1960). Lefschetz also organised numerous congresses and created influential research institutions, from which today's "Lefschetz Center for Dynamical Systems" at Brown University (Providence, RI) arose. This soon led to a steady increase in the number of translations and books in the field, many of them with the adjective "qualitative" in their titles (for example Reissig et al. 1963; Cronin 1980/2007; Reyn 1992; Michel and Wang 1995/2001). Another impulse to western research on the subject was given by A. N. Kolmogorov's renowned lecture at the International Congress of Mathematicians in Amsterdam in 1954, which motivated the profound studies on the stability of dynamical systems by S. Smale, V. I. Arnold and J. Moser, and finally culminated in the KAM theory.[16]

Today, apart from qualitatively oriented problems in topology and modern algebra the qualitative theory of differential equations and dynamical systems is amongst the best-established "qualitative" theories in mathematics. It interacts closely with new qualitatively oriented areas of study, such as catastrophe and chaos theory; their results are used in extra-mathematical fields such as astronomy, physics, chemistry, biology, economics and the human sciences, though with somewhat questionable success. The new theories allow, for example, the modelling of the spontaneous building of a structure, i.e., the self-organisation in animate and inanimate nature, and they demonstrate how in open, unbalanced, nonlinear systems organised structures arise as a result of cooperative inner interaction. In a way, they provide a mathematical model for the "qualitative leap" to another state, as described by Hegel, Marx and Engels.

In physics these qualitative methods were made known by the influential textbook *Foundations of mechanics* (1967, ^{2}1978) by R. Abraham and J. E. Marsden and they subsequently gained importance and interest in other areas of physics (model construction, dimensional analysis, symmetry considerations, phase transitions, critical phenomena). In 1981 M. Gitterman and V. Halpern summarised for the first time in a single work the newly developed methods in their *Qualitative Analysis of Physical Problems*. Their starting point was the statement that most practical physical problems are much too complex to be solved in full generality in a mathematically exact way. Although the physical system in question can in many cases be represented by an adequate system of differential equations, its complete solution is quite often not possible, for either practical or theoretical reasons; in such cases the physically significant quantities have to be estimated by means of so-called qualitative meth-

[16] For further information about the history of the qualitative theory of differential equations and dynamical systems see Aubin and Dahan Dalmedico (2002), Dahan Dalmedico (1994), Gilain (1991), Hirsch (1984), Leimanis (1959), Lyapunov (1992), Petrovitch (1931) and Poincaré (1993). For a detailed listing of Russian contributions in Western review journals see Neuenschwander (2003, p. 30 f., note 21).

ods (Goldstein and Entov 1994, p. 1). Similarly the term "qualitative" can be found in the titles of many other physical works, including Migdal (1977), Villaggio (1977), Bogoyavlensky (1985), Oden (1986), Bakker (1991) and Krainov (1992). Bakker (1991, p. ix) emphasises that a qualitative understanding into physics is of great importance, and even appears as its ultimate aim. It fashions our knowledge and serves as a good guide for further quantitative investigations, especially when it is applied in close correspondence with numerical methods in order to interpret and value numerical results. Migdal (1977, p. xix) has argued that qualitative methods are the ones that constitute the most attractive and beautiful characteristic of theoretical physics, and he regrets that the subject—in contrast with the practices of physical research—is not taught constructively, but is presented rather in a strictly formal, mathematical way.[17]

An interesting addition to these attempts is the latest research in the field of artificial intelligence. In connection with the modelling of common-sense reasoning and its application to the research on physical models, a new field of study has been established, so-called "qualitative physics" or "qualitative reasoning", the results of which are discussed in annual workshops (Weld and de Kleer 1990; Faltings and Struss 1992; etc.). Amongst the main aims of qualitative physics are the qualitative representation of continuous quantities, the formalisation of physical knowledge and the causal analysis of systems. Just as qualitative, "common-sense" based reasoning can be used in tackling physical problems in everyday life, so too in science there are to be found feasible solutions without an exact knowledge of the boundary conditions and the complete resolution of the relevant systems of differential equations. In the qualitative modelling of continuous physical parameters, these parameters are not represented by exact numerical values, but sometimes by the symbols "–", "0", "+", or in the form of intervals, whereas it is not necessary to ascribe a definite numerical value to the interval borders. According to Faltings (1991, p. 40), three basically different approaches have been developed so far for the qualitative modelling of systems until now, namely de Kleer's component-oriented modelling, Kuipers' equation-oriented modelling and Forbus process-oriented modelling (Qualitative Process Theory). According to D'Ambrosio (1989, p. 1); the qualitative descriptions developed in these approaches are important because they provide the ability to reason with incomplete information and can guide the application of more detailed quantitative theories when additional information is available. Furthermore, according to Weld (1990) and Fishwick and Luker (1991), they provide a valuable instrument for the improvement of robots and computers that are capable of learning, while they can also help us to understand how humans reason effectively about complex physical systems. This further opens the possibility of developing intelligent tutoring systems and designing a better user interface between man and machine, in short, of advancing our ability to tackle the translation from qualitative to quantitative knowledge and vice versa.[18]

In spite of the striking rise of so-called qualitative methods in numerous sciences in recent years, it has to be said that the term "qualitative" is used rather as a catchword designating a wide range of very different aspects and approaches that only align with Aristotle's "qualitas"

[17] For a short survey of the qualitative theories in the various areas of mathematics and physics see the articles "Bifurcation", "Chaos", "Dynamical System", "Qualitative Theory of Differential Equations" and "Thom Catastrophes" in Hazewinkel (1988–2002); the article "Qualità/quantità" by Thom (1980) in the *Enciclopedia Einaudi*; Mainzer's and Pechenkin's contributions in Krohn et al. (1992); Atiyah's, Brieskorn's and Thom's contributions in Otte (1974); Scheck (1988, Chap. 6); as well as for example Hirsch et al. (1974/2004), Ott (1993) and Stewart (1989). For additional literature see Neuenschwander (2003, p. 31, note 25).

[18] For a short introduction to the so-called "qualitative physics" or "qualitative reasoning" see for example Bredeweg and Struss (2003), Faltings (1991), and Forbus (1997/2004); for further information see also Bobrow (1984), Dordan (1995), Kuipers (1994), Parsons (2001), Renz (2002), Travé-Massuyès et al. (1997), and Werthner (1994).

in their basic intentions. Hence it is not surprising that even today Rutherford's trenchantly formulated thesis "Qualitative is nothing but poor quantitative" is endorsed by many scientists, although modern epistemological, mathematical and more practical oriented social sciences have gained some new insights. For example R. Carnap and C. G. Hempel have distinguished between qualitative, comparative and quantitative (metric) concepts. Qualitative concepts are defined as class concepts. A field of empirical objects can be structured by qualitative concepts if it allows the introduction of an equivalence relation. Comparative concepts, on the other hand, are ordering concepts. They are introduced over a field of objects by defining an ordering relation for the equivalence classes, which have been created by the equivalence relation. The quantification of a qualitatively given empirical structure indicates a homomorphic mapping onto a numerical structure (K. Mainzer in Ritter and Gründer 1971–2007, vol. 7, column 1825 f.). Thom (1980, p. 464 f.), following Fechner and Riemann, on the other hand, talks about a semantic field of qualities that are continuously transformable into one another. He tries to model this semantic field by means of a hypothetical potential function. And following Nelson Goodman, Clark (1993) argues for the existence of a so-called quality space and he considers the psychological colour solid as its most familiar example. According to Clark (1993, p. 120), this psychological colour solid depends on the observer as well as on the time and is therefore only conditionally objectifiable or intersubjectively describable. Hence one may understand that science tries repeatedly—with the aid of partly questionable reductions—to comprehend "quantitatively" even the "last bastions" of qualitative perception by means of associative scalings and increasingly complex mathematical theories; even though very often just a pale, almost empty framework is left, one that is incapable of comprehending the essential qualities, for example, the beauty of a rose or the euphony of music (Dey 1993, p. 23 ff.; Sorokin 1956, p. 31 ff. and 102 ff.).

In modern analytic philosophy the status of *qualia*—introspectively accessible, phenomenal aspects of our mental lives—is hotly debated because it is central to a proper understanding of the nature of consciousness. Qualia are at the very heart of the mind-body problem. Philosophical disagreement about qualia typically centres on which mental states have qualia, whether qualia are intrinsic qualities of their bearers, and how qualia relate to the physical world both inside and outside the head. Since it is difficult to demonstrate them directly, a more tangential approach is needed. Arguments for qualia generally come in the form of thought experiments designed to lead one to the conclusion that qualia exist. Perhaps the most famous of these is the knowledge argument against physicalism by Frank Jackson about Mary's room. Mary the colour scientist knows all the physical facts about colour. However, she has been confined from birth to a room that is black and white, and is only allowed to observe the outside world through a black and white monitor. When she is finally allowed to leave the room, it must be admitted that she learns something about the colour red the first time she sees it—specifically, she learns what it is like to see that colour. Other arguments for the existence of qualia are the explanatory gap argument by Joseph Levine, the inverted spectrum argument and the zombie argument. So some philosophers believe that qualia are the "cement of the experimental world" (Potrč in Wright 2008, 109 ff.), whereas others such as Daniel Dennett, Paul Churchland, Michael Tye etc. deny their existence.[19]

So in the end it still remains an open question amongst philosophers, whether qualia are irreducible, non-physical entities or if they can be reduced now or at least in future with the development of science to physical origins. Furthermore one may question if the contemporary renaissance of the qualitative in social science is just another episode, which will be

[19] For a short survey of the philosophical theories about qualia see, for example, the relevant articles in the Stanford Encyclopedia of Philosophy or Wikipedia, for further information see Chalmers (2010), Clark (1993), Dennett (1991), Tye (2009), and Wright 2008.

replaced by an even more comprehensive quantification and digitalisation, or if these two rival approaches will unite in a productive synthesis. The latter scenario is supported by the generation of mixed methods in social sciences, that try to integrate quantitative and qualitative approaches. That these have already gained quite a number of proponents is evidenced by the *Journal of Mixed Methods Research* (2007 ff.) and the *Sage Handbook of Mixed Methods in Social & Behavioural Research* (2010).[20] For further information about the multifarious, centuries-long history of quality and quantity, we refer the reader to the monographic article by the well-known mathematician Egbert Brieskorn and the concluding summary by the distinguished philosopher Jürgen Mittelstrass in Neuenschwander (2003). As a complementary study, I would like to point especially to the earlier symposia dedicated to the same subject (Lerner 1961; Glassner and Moreno 1989) and the highly informative articles “Qualität” and “Quantität” in the *Historisches Wörterbuch der Philosophie* (Ritter and Gründer 1971–2007, vol. 7, columns 1748–1780 and 1792–1828) that document the developments from antiquity up to the present day.

References

Abraham, R., Marsden, J.E.: Foundations of Mechanics. A Mathematical Exposition of Classical Mechanics with an Introduction to the Qualitative Theory of Dynamical Systems and Applications to the Three-Body Problem. Benjamin, New York (1967). Second edition: Benjamin/Cummings, Reading (1978)

Adorno, T.W. et al.: Der Positivismusstreit in der deutschen Soziologie. Soziologische Texte, Bd. 58. Luchterhand, Neuwied (1969)

Agarwal, B., Piasetzki, P., Aasen, C.: A Humanistic Approach to Quality of Life: A Selected Bibliography. Exchange Bibliography, Nos. 1052, 1053 and 1054. Council of Planning Librarians, Monticello (1976)

Andronov, A.A.: Les cycles limites de Poincaré et la théorie des oscillations auto-entretenues. Comptes Rendus Hebdomadaires des Séances de l’Académie des Sciences **189**, 559–561 (1929)

Andronov, A.A., Chaikin, C.E.: Theory of Oscillations. English language edition, edited under the direction of S. Lefschetz. Princeton University Press, Princeton (1949)

Andronov, A.A. et al.: Qualitative Theory of Second-Order Dynamic Systems. Translated from Russian by D. Louvish. Wiley, New York (1973)

Arnold, V.I., et al. (eds.): Dynamical Systems I–VIII. Encyclopaedia of Mathematical Sciences, vol. 1–6, 16 and 39. Springer, Berlin (1988–1993)

Aubin, D., Dahan Dalmedico, A.: Writing the history of dynamical systems and chaos: Longue durée and revolution, disciplines and cultures. Historia Mathematica **29**, 273–339 (2002)

Baker, K.M.: Condorcet: From Natural Philosophy to Social Mathematics. The University of Chicago Press, Chicago (1975)

Bakker, P.G.: Bifurcations in Flow Patterns: Some Applications of the Qualitative Theory of Differential Equations in Fluid Dynamics. Nonlinear Topics in the Mathematical Sciences, vol. 2. Kluwer, Dordrecht (1991)

Bässler, R.: Quantitative oder Qualitative Sozialforschung in den Sportwissenschaften. Ein Beitrag zur Methodendiskussion. Methoden der empirischen Sozialforschung in den Sportwissenschaften, Bd. 1. Universitätsverlag für Wissenschaft und Forschung, Wien (1987)

Bissell, C.: A.A. Andronov and the development of Soviet control engineering. IEEE Control Syst. **18**(1), 56–62 (1998)

Blumer, H.: Symbolic Interactionism: Perspective and Method. Prentice-Hall, Englewood Cliffs (1969). Paperback edition: University of California Press, Berkeley (1986)

Bobrow, D.G.: Qualitative Reasoning about Physical Systems. Artificial Intelligence, vol. 24. Elsevier (North-Holland), Amsterdam (1984). Reprint: MIT Press, Cambridge (1985)

Bogdan, R.C., Biklen, S.K.: Qualitative Research for Education: An Introduction to Theory and Methods. Allyn and Bacon, Boston (1982). Fifth edition: Allyn and Bacon, Boston (2006)

[20] Mixed Methods as a third methodological approach in the human sciences started in the late 1980s and 1990s. Amongst its most influential publications belong Bryman (2006), Creswell and Plano Clark (2006/2010), Tashakkori and Teddlie (1998), Tashakkori and Teddlie (2003/2010), and Teddlie and Tashakkori 2009.

Bogoyavlensky, O.I.: Methods in the Qualitative Theory of Dynamical Systems in Astrophysics and Gas Dynamics. Translated from the Russian by D. Gokhman. Springer, Berlin (1985)
Bottomore, T.: The Frankfurt School. Horwood, Chichester (1984)
Botz, G., et al. (Hrsg.): "Qualität und Quantität". Zur Praxis der Methoden der Historischen Sozialwissenschaft. Studien zur Historischen Sozialwissenschaft, Bd. 10. Campus, Frankfurt/New York (1988)
Boudon, R., Cherkaoui, M., Demeulenaere, P.: The European Tradition in Qualitative Research, 4 vols. Sage, London (2003)
Bredeweg, B., Struss, P. (eds.): Qualitative Reasoning. AI Mag. **24**(4) (2003)
Bryant, C.G.A.: Positivism in Social Theory and Research. St. Martin's Press, New York (1985)
Bryman, A.: Quantity and Quality in Social Research. Unwin Hyman, London (1988)
Bryman, A., Burgess, R.G. (eds.): Qualitative Research, 4 vols. Sage, London (1999). Complementary edition by A. Bryman: Qualitative Research 2. Sage, London (2007)
Bryman, A. (ed.): Mixed Methods, 4 vols. Sage, London (2006)
Bulmer, M.: The Chicago School of Sociology. Institutionalization, Diversity, and the Rise of Sociological Research. The University of Chicago Press, Chicago (1984)
Campbell, A., Converse, P.E., Rodgers, W.L.: The Quality of American Life: Perceptions, Evaluations, and Satisfactions. Russell Sage Foundation, New York (1976)
Carnap, R.: Die physikalische Sprache als Universalsprache der Wissenschaft. Erkenntnis **2**, 432–465 (1931)
Chalmers, D.J.: The Character of Consciousness. Oxford University Press, Oxford (2010)
Clagett, M.: The Science of Mechanics in the Middle Ages. The University of Wisconsin Publications in Medieval Science, vol. 4. The University of Wisconsin Press, Madison (1959)
Clagett, M. (ed.): Nicole Oresme and the Medieval Geometry of Qualities and Motions. A treatise on the uniformity and difformity of intensities known as "Tractatus de configurationibus qualitatum et motuum". Edited with an introduction, English translation, and commentary. The University of Wisconsin Publications in Medieval Science, vol. 12. The University of Wisconsin Press, Madison (1968)
Clark, A.: Sensory Qualities. Clarendon Press, Oxford (1993)
Club of Rome (Hrsg.): Die Herausforderung des Wachstums. Globale Industrialisierung: Hoffnung oder Gefahr? Zur Lage der Menschheit am Ende des Jahrtausends. Berichte internationaler Experten an den Club of Rome. Scherz, Bern (1990)
Cohen, I.B. (ed.): The Natural Sciences and the Social Sciences: Some Critical and Historical Perspectives. Boston Studies in the Philosophy of Science, vol. 150. Kluwer, Dordrecht (1994)
Cook, T.D., Reichardt, C.S. (eds.): Qualitative and Quantitative Methods in Evaluation Research. Sage Research Progress Series in Evaluation, vol. 1. Sage, Beverly Hills (1979)
Creswell, J.W., Plano Clark, V.L.: Designing and Conducting Mixed Methods Research. Sage, Thousand Oaks (2006). Second edition: Sage (2010)
Crombie, A.C.: Styles of Scientific Thinking in the European Tradition: The History of Argument and Explanation Especially in the Mathematical and Biomedical Sciences and Arts, 3 vols. Duckworth, London (1994)
Cronin, J.: Differential Equations. Introduction and Qualitative Theory. Dekker, New York (1980). Third edition: Chapman & Hall/CRC, Boca Raton (2007)
Dahan Dalmedico, A.: La renaissance des systèmes dynamiques aux Etats-Unis après la deuxième guerre mondiale: l'action de Solomon Lefschetz. Rendiconti del Circolo Matematico di Palermo, Serie II, Supplemento No. 34, pp. 133–166 (1994)
Dahms, H.-J.: Positivismusstreit. Die Auseinandersetzungen der Frankfurter Schule mit dem logischen Positivismus, dem amerikanischen Pragmatismus und dem kritischen Rationalismus. Suhrkamp Taschenbuch Wissenschaft, Bd. 1058. Suhrkamp, Frankfurt am Main (1994)
D'Ambrosio, B.: Qualitative Process Theory Using Linguistic Variables. Springer, New York (1989)
Dennett, D.C.: Consciousness Explained. Lane, London (1991)
Denzin, N.K., Lincoln, Y.S. (eds.): Handbook of Qualitative Research. Sage, Thousand Oaks (1994). Third edition: Sage, Thousand Oaks (2005)
Denzin, N.K., Lincoln, Y.S. (eds.): The American Tradition in Qualitative Research, 4 vols. Sage, London (2001)
Dey, I.: Qualitative Data Analysis: A User-Friendly Guide for Social Scientists. Routledge, London (1993)
Dijksterhuis, E.J.: De mechanisering van het wereldbeeld. Meulenhoff, Amsterdam (1950). English Translation: The Mechanization of the World Picture. Clarendon Press, Oxford (1961). Reprint: Princeton University Press, Princeton (1986)
Dordan, O.: Analyse Qualitative. Masson, Paris (1995)
Ehrlich, P.R.: The Population Bomb. Ballantine, New York (1968). Revised edition: Ballantine, New York (1971)
Faltings, B.: Wissensrepräsentation und qualitatives Schliessen. In: Struss, P. (Hrsg.) Wissensrepräsentation, pp. 33–41. Oldenbourg, München (1991)

Faltings, B., Struss, P. (eds.): Recent Advances in Qualitative Physics. MIT Press, Cambridge (1992)
Faris, R.E.L.: Chicago Sociology: 1920–1932. Chandler Publishing Company, San Francisco (1967). Reprinted with a foreword by M. Janowitz. The University of Chicago Press, Chicago (1970)
Fishwick, P.A., Luker, P.A. (eds.): Qualitative Simulation Modeling and Analysis. Advances in Simulation, vol. 5. Springer, New York (1991)
Flick, U., et al. (eds.): Handbuch qualitative Sozialforschung. Grundlagen, Konzepte, Methoden und Anwendungen. Psychologie Verlags Union, München (1991). Revised edition: Qualitative Forschung: ein Handbuch. Rowohlt, Reinbek (2000). Sixth edition: 2008. English edition: A Companion to Qualitative Research. Sage, London (2004)
Flick, U.: Qualitative Forschung. Theorie, Methoden, Anwendung in Psychologie und Sozialwissenschaften. Rowohlt, Reinbek (1995). Revised editions: Qualitative Sozialforschung, eine Einführung. 2002 ff. English edition: An Introduction to Qualitative Research. Sage, London (1999). Fourth edition: Sage, London (2009)
Forbus, K.D.: Qualitative Reasoning. In: Tucker, A.B., Jr. (ed.) The Computer Science and Engineering Handbook, Chap. 32. CRC Press, Boca Raton (1997). Second edition: Chapman & Hall/CRC, Boca Raton, Chap. 62 (2004)
Forrester, J.W.: World Dynamics. Wright-Allen, Cambridge (1971)
Frängsmyr, T., Heilbron, J.L., Rider, R.E. (eds.): The Quantifying Spirit in the 18th Century. Uppsala Studies in History of Science, vol. 7. University of California Press, Berkeley (1990)
Friedrichs, G. (Hrsg.): Aufgabe Zukunft: Qualität des Lebens. Beiträge zur vierten internationalen Arbeitstagung der Industriegewerkschaft Metall für die Bundesrepublik Deutschland 11. bis 14. April 1972 in Oberhausen, 10 Bde. Europäische Verlagsanstalt, Frankfurt am Main (1973–1974)
Gabor, D. et al.: Beyond the Age of Waste. A Report to the Club of Rome. Pergamon Press, Oxford (1978)
Gage, N.L.: The paradigm wars and their aftermath: A “historical” sketch of research on teaching since 1989. Educ. Res. **18**(7), 4–10 (1989)
Galbraith, J.K.: The Affluent Society. Hamish Hamilton, London (1958); 40th Anniversary edition: Houghton Mifflin, New York (1998)
Garfinkel, H.: Studies in Ethnomethodology. Prentice-Hall, Englewood Cliffs (1967)
Garz, D., Kraimer, K. (Hrsg.): Qualitativ-empirische Sozialforschung. Konzepte, Methoden, Analysen. Westdeutscher Verlag, Opladen (1991)
Gilain, C.: La théorie qualitative de Poincaré et le problème de l’intégration des équations différentielles. In: Gispert, H. (ed.) La France mathématique. La Société mathématique de France (1872–1914). Cahiers d’Historie et de Philosophie des Sciences, Nouvelle Série, No. 34, pp. 215–242. Paris (1991)
Gitterman, M., Halpern, V.: Qualitative Analysis of Physical Problems. Academic Press, New York (1981)
Glaser, B.G., Strauss, A.L.: The Discovery of Grounded Theory: Strategies for Qualitative Research. Aldine de Gruyter, New York (1967)
Glassner, B., Moreno, J.D. (eds.): The Qualitative-Quantitative Distinction in the Social Sciences. Boston Studies in the Philosophy of Science, vol. 112. Kluwer, Dordrecht (1989)
Goldstein, R.V., Entov, V.M.: Qualitative Methods in Continuum Mechanics. Pitman Monographs and Surveys in Pure and Applied Mathematics, vol. 72. Longman, Harlow (1994)
Goodman, N.: The Structure of Appearance. Harvard University Press, Cambridge (1951). Third edition with an introduction by G. Hellman. Boston Studies in the Philosophy of Science, vol. 53 = Synthese Library, vol. 107. Reidel, Dordrecht (1977)
Grant, E.: Physical Science in the Middle Ages. Cambridge University Press, Cambridge (1977)
Greenwood, T.: Les mathématiques qualitatives. Revue Trimestrielle Canadienne **37**, 287–308 (1951)
Guth, S.: Chicago 1920: aux origines de la sociologie qualitative. Téraèdre, Paris (2004)
Halfpenny, P.: Positivism and Sociology: Explaining Social Life. Controversies in Sociology, vol. 13. Allen & Unwin, London (1982)
Halfpenny, P., McMylor, P. (eds.): Positivist Sociology and its Critics, 3 vols. Schools of Thought in Sociology, vol. 11. Elgar, Aldershot (1994)
Hammersley, M.: The Dilemma of Qualitative Method: Herbert Blumer and the Chicago Tradition. Routledge, London (1989). Reprinted in paperback: Routledge, London (1990)
Hammersley, M.: Questioning Qualitative Inquiry. Critical Essays. Sage, London (2008)
Hazewinkel, M. (ed.): Encyclopaedia of Mathematics. An updated and annotated translation of the Soviet ‘Mathematical Encyclopaedia’, 10 vols. Reidel, Dordrecht (1988–1994). Supplement I–III (1997–2002)
Herfindahl, O.C., Kneese, A.V.: Quality of the Environment: An Economic Approach to Some Problems in Using Land, Water, and Air. Resources for the Future, Washington, DC (1965)
Hirsch, M.W.: The dynamical systems approach to differential equations. Bull. Am. Math. Soc. **11**, 1–64 (1984)
Hirsch, M.W., et al.: Differential Equations, Dynamical Systems, and Linear Algebra. Academic Press, New York (1974). Second revised edition: Elsevier, Amsterdam (2004)

Israel, G.: Technological innovation and new mathematics: van der Pol and the birth of nonlinear dynamics. In: Lucertini, M., et al. (eds.) Technological Concepts and Mathematical Models in the Evolution of Modern Engineering Systems, pp. 52–78. Birkhäuser, Basel (2004)
Jacob, E.: Qualitative research traditions: a review. Rev. Educ. Res. **57**, 1–50 (1987)
Jarrett, H. (ed.): Environmental Quality in a Growing Economy. Resources for the Future, Washington, DC (1966)
Kern, H.: Empirische Sozialforschung. Ursprünge, Ansätze, Entwicklungslinien. Beck, München (1982)
Kneese, A.V., Bower, B.T.: Managing Water Quality: Economics, Technology, Institutions. Resources for the Future, Washington, DC (1968). Paperback edition: Resources for the Future, Washington, DC (1984)
Krainov, V.P.: Qualitative Methods in Physical Kinetics and Hydrodynamics (trans: K. Hendzel). American Institute of Physics, New York (1992)
Krohn,W., Krug, H.-J., Küppers, G. (Hrsg.): Konzepte von Chaos und Selbstorganisation in der Geschichte der Wissenschaften. Selbstorganisation: Jahrbuch für Komplexität in den Natur-, Sozial- und Geisteswissenschaften, Bd. 3. Duncker & Humblot, Berlin (1992)
Krüger, L., et al. (eds.): The Probabilistic Revolution, 2 vols. MIT Press, Cambridge (1987)
Kryloff, N., Bogoliuboff, N.: Introduction to Non-Linear Mechanics. A free translation by Solomon Lefschetz of excerpts from two Russian monographs. Annals of Mathematics Studies, No. 11. Princeton University Press, Princeton (1943)
Kuhn, T.S.: The Copernican Revolution: Planetary Astronomy in the Development of Western Thought. Harvard University Press, Cambridge (1957)
Kuhn, T.S.: The Structure of Scientific Revolutions. The University of Chicago Press, Chicago (1962). Third edition: Chicago (1996)
Kuipers, B.: Qualitative Reasoning: Modeling and Simulation with Incomplete Knowledge. MIT Press, Cambridge (1994)
Lamnek, S.: Qualitative Sozialforschung, 2 Bde. Psychologie Verlags Union, München (1988–1989). 4., vollständig überarbeitete Auflage: Beltz, Weinheim (2005)
Lancaster, H.O.: Quantitative Methods in Biological and Medical Sciences: A Historical Essay. Springer, New York (1994)
Lang, H.S.: Aristotle's Physics and its Medieval Varieties. State University of New York Press, Albany (1992)
Lazarsfeld, P.F.: Qualitative Analysis: Historical and Critical Essays. Allyn and Bacon, Boston (1972)
Leimanis, E.: Qualitative methods in general dynamics and celestial mechanics. Applied Mechanics Reviews **12**, 665–670 (1959)
Lerner, D. (ed.): Quantity and Quality. The Hayden Colloquium on Scientific Method and Concept. The Free Press of Glencoe, New York (1961)
Lyapunov, A.M.: The General Problem of the Stability of Motion. Translated and edited by A.T. Fuller. With a biography of Lyapunov by V.I. Smirnov and a bibliography of Lyapunov's works by J.F. Barrett. Taylor & Francis, London (1992)
Malinowski, B.: Argonauts of the Western Pacific: An Account of Native Enterprise and Adventure in the Archipelagoes of Melanesian New Guinea. Routledge, London (1922)
Mayring, P.: Qualitative Inhaltsanalyse. Grundlagen und Techniken. Beltz, Weinheim (1983). 10., neu ausgestattete Auflage, Beltz, Weinheim (2008)
Mayring, P.: Einführung in die qualitative Sozialforschung. Eine Anleitung zu qualitativem Denken. Psychologie Verlags Union, München (1990). 5., überarbeitete und neu ausgestattete Auflage: Beltz, Weinheim (2002)
Meadows, D.H., et al.: The Limits to Growth. A Report for the Club of Rome's Project on the Predicament of Mankind. Universe, New York (1972). The 30-year update: Earthscan, London (2005)
Meadows, D.H., Meadows, D.L., Randers, J.: Beyond the Limits: Global Collapse or a Sustainable Future. Chelsea Green Publishing Co., Post Mills (1992)
Meadows, D.L. et al.: Dynamics of Growth in a Finite World. Wright-Allen, Cambridge (1974)
Merwin, D.J.: The Quality of Life: A Bibliography of Objective and Perceptual Social Indicators. Council of Planning Librarians. Exchange Bibliography, No. 1079. Council of Planning Librarians, Monticello (1976)
Michel, Anthony N., Wang, Kaining: Qualitative Theory of Dynamical Systems: The Role of Stability Preserving Mappings. Monographs and Textbooks in Pure and Applied Mathematics, vol. 186. Dekker, New York (1995). Second edition, revised and expanded: Dekker, New York (2001)
Migdal, A.B.: Qualitative Methods in Quantum Theory. Translated from the Russian edition by A.J. Leggett. Frontiers in Physics, No. 48. Benjamin, Reading (1977). Reprint: Addison-Wesley, Redwood City (1989)
Mishan, E.J.: The Costs of Economic Growth. Staples Press, London (1967). New, revised edition: Weidenfeld and Nicolson, London (1993)
Mishan, E.J.: The Economic Growth Debate: An Assessment. Allen & Unwin, London (1977)

National Goals Research Staff: Toward Balanced Growth: Quantity with Quality. Report of the National Goals Research Staff. U.S. Government Printing Office, Washington, DC (1970)
Nemytskii, V.V., Stepanov, V.V.: Qualitative Theory of Differential Equations. English language edition, edited under the direction of S. Lefschetz. Princeton Mathematical Series, vol. 22. Princeton University Press, Princeton (1960). Reprint: Dover, New York (1989)
Neuenschwander, E. (Hrsg.): Wissenschaft, Gesellschaft und politische Macht. Birkhäuser, Basel (1993)
Neuenschwander, E. (Hrsg.): Wissenschaft zwischen Qualitas und Quantitas. Birkhäuser, Basel (2003)
Noll, H.-H.: Beschäftigungschancen und Arbeitsbedingungen. Ein Sozialbericht für die Bundesrepublik 1950–1980. Sonderforschungsbereich 3 der Universitäten Frankfurt und Mannheim "Mikroanalytische Grundlagen der Gesellschaftspolitik", Schriftenreihe Band 9. Campus, Frankfurt (1982)
Oden, J.T.: Qualitative Methods in Nonlinear Mechanics. Prentice-Hall, Englewood Cliffs (1986)
Otte, M. (Hrsg.): Mathematiker über die Mathematik. Springer, Berlin (1974)
Ott, E.: Chaos in Dynamical Systems. Cambridge University Press, Cambridge (1993). Second edition: Cambridge (2002)
Parsons, S.: Qualitative Methods for Reasoning under Uncertainty. MIT Press, Cambridge (2001)
Patel, N.: Quantitative and collaborative trends in American sociological research. Am. Sociol. **7**(9), 5–6 (1972)
Pedersen, O., Pihl, M.: Early Physics and Astronomy. A Historical Introduction. Macdonald and Janes, London (1974). Second, revised edition: Cambridge University Press, Cambridge (1993)
Pestel, E.: Beyond the Limits to Growth: A Report to the Club of Rome. Universe Books, New York (1989)
Petrovitch, M.: Intégration qualitative des équations différentielles. Mémorial des Sciences Mathématiques, No. 48. Gauthier-Villars, Paris (1931)
Pirsig, R.M.: Zen and the Art of Motorcycle Maintenance: An Inquiry into Values. William Morrow, New York (1974). 25th Anniversary Edition: Random House, London (1999)
Poincaré, H.: Mémoire sur les courbes définies par une équation différentielle. Journal de Mathématiques pures et appliquées (3) **7** (1881), 375–422; **8** (1882), 251–296; (4) **1** (1885), 167–244; **2** (1886), 151–217 = *Œuvres de Henri Poincaré*, vol. 1. Gauthier-Villars, Paris, 1928. 3–84, 90–158, 167–222
Poincaré, H.: New Methods of Celestial Mechanics, 3 vols. Edited and introduced by D.L. Goroff. History of Modern Physics and Astronomy, vol. 13. American Institute of Physics, New York (1993)
Pólya, G.: Mathematical Discovery: On Understanding, Learning, and Teaching Problem Solving, 2 vols. Wiley, New York (1962–1965)
Porter, T.M.: The Rise of Statistical Thinking, 1820–1900. Princeton University Press, Princeton (1986)
Porter, T.M.: Trust in Numbers. The Pursuit of Objectivity in Science and Public Life. Princeton University Press, Princeton (1995)
Porter, T.M., Ross, D. (eds.): The Modern Social Sciences. The Cambridge History of Science, vol. 7. Cambridge University Press, Cambridge (2003)
Pretty, J., et al. (eds.): The SAGE Handbook of Environment and Society. Sage, London (2007)
Quetelet, A.: Instructions populaires sur le calcul des probabilités. Tarlier/Hayez, Bruxelles (1828)
Radford, J., et al.: Quantity and Quality in Higher Education. Higher Education Policy Series 40. Jessica Kingsley, London (1997)
Rapoport, A.: Mathematische Methoden in den Sozialwissenschaften. Physica-Verlag, Würzburg (1980). English edition: Mathematical Models in the Social and Behavioral Sciences. Wiley, New York (1983)
Rashed, R. (ed.): Condorcet: Mathématique et société. Hermann, Paris (1974)
Reissig, R., Sansone, G., Conti, R.: Qualitative Theorie nichtlinearer Differentialgleichungen. Cremonese, Roma (1963)
Renz, J.: Qualitative Spatial Reasoning with Topological Information. Springer, Berlin (2002)
Rescher, N. (ed.): The Heritage of Logical Positivism. University Press of America, Lanham (1985)
Reyn, J.W.: A Bibliography of the Qualitative Theory of Quadratic Systems of Differential Equations in the Plane. Second edition: Reports of the Faculty of Technical Mathematics and Informatics, no. 92-17. Technische Universiteit Delft, Delft (1992)
Ritter, J., Gründer, K., et al. (Hrsg.): Historisches Wörterbuch der Philosophie, 13 vols. Schwabe, Basel (1971–2007)
Rizo, F.M.: The controversy about quantification in social research: an extension of Gage's "'Historical' Sketch". Educ. Res. **20**(9), 9–12 (1991)
Sahner, H.: Theorie und Forschung. Zur paradigmatischen Struktur der westdeutschen Soziologie und zu ihrem Einfluss auf die Forschung. Beiträge zur sozialwissenschaftlichen Forschung, Bd. 34. Westdeutscher Verlag, Opladen (1982)
Salomon, G.: Transcending the qualitative-quantitative debate: the analytic and systemic approaches to educational research. Edu. Res. **20**(6), 10–18 (1991)

Scheck, F.: Mechanik. Von den Newtonschen Gesetzen zum deterministischen Chaos. Springer, Berlin (1988). 8. Auflage: Springer, Berlin (2007). English translation, fifth edition: Mechanics: From Newton's Laws to Deterministic Chaos. Springer, Berlin (2010)
Schulte, J., McGuinness, B. (Hrsg.): Einheitswissenschaft. Suhrkamp Taschenbuch Wissenschaft, Bd. 963. Suhrkamp, Frankfurt am Main (1992)
Schultz, U. (Hrsg.): Lebensqualität. Konkrete Vorschläge zu einem abstrakten Begriff. Aspekte Verlag, Frankfurt am Main (1975)
Schumpeter, J.A.: History of Economic Analysis. Edited from manuscript by E.B. Schumpeter. Oxford University Press, New York (1954)
Seifert, G. (Hrsg.): Lebensqualität in unserer Zeit – Modebegriff oder neues Denken?–. Veröffentlichung der Joachim Jungius-Gesellschaft der Wissenschaften Hamburg, Nr. 69. Vandenhoeck & Ruprecht, Göttingen (1992)
Shea, W.R. (ed.): Nature Mathematized: Historical and Philosophical Case Studies in Classical Modern Natural Philosophy. Papers Deriving from the Third International Conference on the History and Philosophy of Science, Montreal, Canada, 1980. The University of Western Ontario Series in Philosophy of Science, vol. 20. Reidel, Dordrecht (1983)
Smith, J.A. (ed.): Qualitative Psychology. A Practical Guide to Research Methods. Sage, London (2003). Second edition: Sage, London (2008)
Sorokin, P.A.: Fads and Foibles in Modern Sociology and Related Sciences. Henry Regnery Company, Chicago (1956)
Stadler, F. (ed.): Scientific Philosophy: Origins and Developments. Vienna Circle Institute Yearbook, vol. 1. Kluwer, Dordrecht (1993)
Stark, H.: Panzer – Qualität oder Quantität. Die Mittelwertmethode zur Bewertung der Panzertechnologie der Zukunft, 2 Bde. Bernard & Graefe Verlag, München (1982)
Stewart, I.: Does God Play Dice? The Mathematics of Chaos. Basil Blackwell, Oxford (1989). Second edition: Blackwell 1997
Stigler, S.M.: The History of Statistics: The Measurement of Uncertainty Before 1900. The Belknap Press of Harvard University Press, Cambridge (1986)
Strauss, A.L.: Qualitative Analysis for Social Scientists. Cambridge University Press, Cambridge (1987)
Strauss, A.L., Corbin, J.: Basics of Qualitative Research: Grounded Theory Procedures and Techniques. Sage, Newbury Park (1990). Third edition: Sage, Thousand Oaks (2008)
Sylla, E.D.: Medieval quantifications of qualities: The "Merton School". Arch. Hist. Exact Sci. **8**, 9–39 (1971/1972)
Sylla, E.D.: The Oxford Calculators and the Mathematics of Motion 1320–1350: Physics and Measurement by Latitudes. Harvard Dissertations in the History of Science. Garland, New York (1991)
Szalai, A., Andrews, F.M. (eds.): The Quality of Life: Comparative Studies. Sage Studies in International Sociology, vol. 20. Sage, London (1980)
Tashakkori, A., Teddlie, C.: Mixed Methodology: Combining Qualitative and Quantitative Approaches. Sage, Thousand Oaks (1998)
Tashakkori, A., Teddlie, C. (eds.): Handbook of Mixed Methods in Social & Behavioral Research. Sage, Thousand Oaks (2003). Second edition: Sage, Thousand Oaks (2010)
Teddlie, C., Tashakkori, A.: Foundations of Mixed Methods Research: Integrating Quantitative and Qualitative Approaches in the Social and Behavioral Sciences. Sage, Thousand Oaks (2009)
Thom, R.: Qualità/quantità. In: Enciclopedia, vol. 11, pp. 460–476. Einaudi, Torino (1980)
Thom, R.: Esquisse d'une sémiophysique. Physique aristotelicienne et théorie des catastrophes. InterEditions, Paris (1988). English Translation: Semio Physics: A Sketch. Aristotelian Physics and Catastrophe Theory. Addison-Wesley, Redwood City (1990)
Tolman, C.W. (ed.): Positivism in Psychology: Historical and Contemporary Problems. Springer, New York (1992)
Travé-Massuyès, L., Dague, P., Guerrin, F.: Le raisonnement qualitatif pour les sciences de l'ingénieur. Hermes, Paris (1997)
Tuschling, B., Rischmüller, M.: Kritik des Logischen Empirismus. Sozialwissenschaftliche Schriften, Heft 6. Duncker & Humblot, Berlin (1983)
Tye, M.: Consciousness Revisited: Materialism without Phenomenal Concepts. MIT Press, Cambridge (2009)
Villaggio, P.: Qualitative Methods in Elasticity. Monographs and Textbooks on Mechanics of Solids and Fluids. Mechanics of Continua, vol. 3. Noordhoff, Leyden (1977)
Wassmuth, R., Sarican, C.: Qualität und Quantität der Lammfleischerzeugung in Westanatolien unter besonderer Berücksichtigung der Einkreuzung von Ostfriesischen Milchschafen. Giessener Schriftenreihe Tierzucht und Haustiergenetik, Bd. 46. Parey, Hamburg (1984)
Weld, D.S.: Theories of Comparative Analysis. MIT Press, Cambridge (1990)

Weld, D.S., de Kleer, J. (eds.): Readings in Qualitative Reasoning about Physical Systems. Morgan Kaufmann, San Mateo (1990)

Werthner, H.: Qualitative Reasoning: Modeling and the Generation of Behavior. Springer, Wien (1994)

Westfall, R.S.: The Construction of Modern Science: Mechanisms and Mechanics. Wiley, New York (1971)

Witzel, A.: Verfahren der qualitativen Sozialforschung. Überblick und Alternativen. Campus Forschung, Bd. 322. Campus, Frankfurt (1982)

Wolff, M.: Geschichte der Impetustheorie. Untersuchungen zum Ursprung der klassischen Mechanik. Suhrkamp, Frankfurt am Main (1978)

Wolfgang, M.E. (ed.): The Environment and the Quality of Life: A World View. The Annals of the American Academy of Political and Social Science, vol. 444. The American Academy of Political and Social Science, Philadelphia (1979)

Woolf, H. (ed.): Proceedings of the Conference on the History of Quantification in the Sciences. [Papers on quantification in science: general aspects, medieval physics, modern physical science, chemistry, medical science, psychology, economics, sociology and biology, by S.S. Wilks, A.C. Crombie, T.S. Kuhn et al.]. Isis **52**, No. 168 (1961)

Wright, E. (ed.): The Case for Qualia. MIT Press, Cambridge (2008)

Zedler, P., Moser, H. (Hrsg.): Aspekte qualitativer Sozialforschung. Studien zu Aktionsforschung, empirischer Hermeneutik und reflexiver Sozialtechnologie. Leske + Budrich, Opladen (1983)